Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 4C Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	Moments about y-axis	M1	3.1a
	$2a(7+k)=3\times 2a+4\times a+k\times 2\mu a$ $(14+2k=10+2\mu k)$	A1	1.1b
	Moments about x-axis	M1	3.1a
	$4a(7+k)=3\times 3a+4\times 5a+k\times \mu a$ $(28+4k=29+\mu k)$	A1	1.1b
	Obtain $k=1$ or $\mu=3$	A1	2.2a
	Obtain $k=1$ and $\mu=3$	A1	2.2a
		(6)	
			(6 marks)

Notes:

M1	Dimensionally correct equation with all terms. Allow with $\bar{x}$ in place of $2a$
A1	Correct unsimplified equation.
M1	Dimensionally correct equation with all terms. Allow with $\bar{y}$ in place of $4a$
A1	Correct unsimplified equation.
	These two equations can be combined as a single vector equation.
A1	One correct value.
A1	Both correct values.

Question	Scheme	Marks	AOs
2(a)	$2\dfrac{\mathrm{d}v}{\mathrm{d}t}=2+v$	M1	2.1
	$\Rightarrow\int\dfrac{2}{2+v}\mathrm{d}v=\int 1\mathrm{d}t$	DM1	1.1b
	$2\ln\lvert 2+v\rvert=t\,(+c)$	A1	1.1b
	$T=2\ln 12-2\ln 7=2\ln\dfrac{12}{7}$ *	A1*	2.2a
		(4)	
2(a) alt	$\dfrac{\mathrm{d}v}{\mathrm{d}t}-\dfrac{1}{2}v=1$	M1	
	Use integrating factor $\mathrm{e}^{-\frac{1}{2}t}$ and integrate	DM1	
	Obtain $v\mathrm{e}^{-\frac{1}{2}t}=-2\mathrm{e}^{-\frac{1}{2}t}\,(+C)$	A1	
	$\Rightarrow v=-2+7\mathrm{e}^{\frac{1}{2}t}$, $T=2\ln\dfrac{12}{7}$ *	A1*	
		(4)	
2(a) alt	$\dfrac{\mathrm{d}v}{\mathrm{d}t}-\dfrac{1}{2}v=1$	M1	
	Auxiliary equation is $\lambda-\dfrac{1}{2}=0$ so CF is $v=A\mathrm{e}^{\frac{1}{2}t}$ and PI is $v=-2$	DM1	
	$v=A\mathrm{e}^{\frac{1}{2}t}-2$	A1	
	$t=0, v=5\rightarrow A=7\Rightarrow 10=7\mathrm{e}^{\frac{1}{2}T}-2\Rightarrow T=2\ln\dfrac{12}{7}$	A1	
		(4)	
(b)	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=1+\dfrac{1}{2}v$	M1	3.4
	$\Rightarrow\int\dfrac{2v}{2+v}\mathrm{d}v=\int 1\mathrm{d}x=\int 2-\dfrac{4}{2+v}\mathrm{d}v$	DM1	1.1b
	$x\,(+A)=2v-4\ln\lvert 2+v\rvert$	A1	1.1b
	$D=2(10-5)-4\ln 12+4\ln 7=10-4\ln\left(\dfrac{12}{7}\right)$	A1	1.1b
		(4)	
(b) alt	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=-2+7\mathrm{e}^{\frac{1}{2}t}$	M1	

	Integrate	DM1	
	Obtain $x = -2t + 14e^{\frac{1}{2}t} (+A)$	A1	
	$D = -4\ln\left(\frac{12}{7}\right) + 14 \times \frac{12}{7} - 14 = 10 - 4\ln\left(\frac{12}{7}\right)$	A1	
		(4)	
(b) alt	$2\ddot{x} - \dot{x} = 2$	M1	
	AE: $2m^2 - m = 0$, CF: $x = Ae^{\frac{1}{2}t} + B$, PI: $x = -2t$	M1	
	$x = Ae^{\frac{1}{2}t} + B - 2t$	A1	
	$t = 0, x = 0, \dot{x} = 5 \Rightarrow x = 14e^{\frac{1}{2}t} - 14 - 2t \Rightarrow D = 10 - 4\ln\left(\frac{12}{7}\right)$	A1	
		(4)	

(8 marks)

Notes:

(a) M1	Obtain a differential equation in v and t Allow $\pm$ for the acceleration
M1	Separate variables and integrate.
A1	Or equivalent unsimplified form. Allow without modulus signs.
A1*	Use limits to eliminate constant of integration, or in a definite integral, to obtain given answer from full and correct working.
(b) M1	Obtain a differential equation in v and x Allow $\pm$ for the acceleration
M1	Separate variables and integrate. If using integration by parts they need to complete the integration to score M1.
A1	Or equivalent unsimplified form. Allow without modulus signs.
A1	Use limits to eliminate constant of integration, or in a definite integral, to obtain exact distance from exact working. Any equivalent simplified form.

Question	Scheme	Marks	AOs
3(a)	$\alpha = \frac{\pi}{4} \Rightarrow \frac{r\sin\alpha}{\alpha} = r \times \frac{1}{\sqrt{2}} \times \frac{4}{\pi} \left(= \frac{2\sqrt{2}r}{\pi} \right)$	B1	1.1b
	Distance from $OA = \frac{2\sqrt{2}r}{\pi} \times \cos\frac{\pi}{4} = \frac{2\sqrt{2}r}{\pi \times \sqrt{2}} = \frac{2r}{\pi}$ *	B1*	2.2a
		(2)	
3(b)	Moments about *OA*:	M1	3.1a
	$r \times \frac{r}{2} + r \times r + \frac{2r}{\pi} \times \frac{\pi r}{2} + \frac{2r}{\pi} \times \frac{\pi r}{2} = (3r + \pi r)d$	A1 A1	1.1b 1.1b
	$\left(\frac{7r^2}{2} = (3r + \pi r)d \right) \Rightarrow d = \frac{7r}{2(3+\pi)}$ *	A1*	2.2a
		(4)	
3(c)	Moments about *A* or any other complete method to obtain *F* in terms of *W*	M1	3.1a
	$W \times \frac{7r}{2(3+\pi)} = F \times 2r$	A1	1.1b
	$F = \frac{7}{4(3+\pi)}W$	A1	1.1b
		(3)	
			(9 marks)

Notes:	
(a)B1	Correct use of given formula. Seen or implied.
B1*	Obtain given result from correct working, e.g. by use of trigonometry or use of Pythagoras.
(b)M1	Condone a dimension error in the arc length, but otherwise must be a dimensionally correct equation. Need all terms. Allow use of a parallel axis.
A1 **A1**	Unsimplified equation with at most one error. Correct unsimplified equation. An error in the arc length affects 3 terms – count it as a single error.
A1*	Obtain given answer from correct working.
(c)M1	Dimensionally correct equation with distances perpendicular to forces..
A1	Correct unsimplified equation.

A1	Any equivalent form. $0.28W$ or better $(0.28494...W)$

Question	Scheme	Marks	AOs
4(a)	Conservation of energy:	M1	3.1a
	$\frac{1}{2}Mv^2 = Mga(1-\cos\theta)$	A1	1.1b
	Equation of motion:	M1	3.1a
	$\frac{Mv^2}{a} = Mg\cos\theta - R$	A1	1.1b
	Solve for R:	DM1	2.1
	$R = Mg\cos\theta - \frac{Mv^2}{a} = Mg\cos\theta - 2Mg(1-\cos\theta)$ $= 3Mg\cos\theta - 2Mg = Mg(3\cos\theta - 2)$ *	A1*	2.2a
		(6)	
(b)	$R = 0 \Rightarrow \cos\theta = \frac{2}{3}$	B1	1.1b
	$v^2 = 2ga(1-\cos\theta)$	M1	3.1a
	$v^2 = \frac{2}{3}ga, \quad v = \sqrt{\frac{2ga}{3}}$	A1	1.1b
		(3)	
			(9 marks)

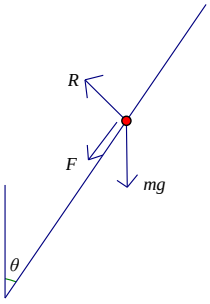
Notes:	
(a) M1	Dimensionally correct equation. All relevant terms. Condone sign error(s) and sin/cos confusion.
A1	Correct unsimplified equation.
M1	Dimensionally correct equation. All relevant terms. Condone sign error(s) and sin/cos confusion.
A1	Correct unsimplified equation.
	If they have more than 2 equations, mark the correct equations. If they go on to use an incorrect equation then DM0.
DM1	Complete method to obtain expression for R
A1*	Obtain given answer from full and correct working.
(b) B1	Seen or implied.

M1	Complete method to obtain v or v^2
A1	Any equivalent form. Allow $0.82\sqrt{ga}$ or better.

Question	Scheme	Marks	AOs
5a	$\int \frac{1}{2}y^2\,\mathrm{d}x = \frac{1}{2}\int \left(9-x^2\right)^2\,\mathrm{d}x \left(= \frac{1}{2}\int \left(81-18x^2+x^4\right)\mathrm{d}x\right)$	M1	2.1
	$= \frac{1}{2}\left[81x-6x^3+\frac{1}{5}x^5\right]_0^3$	DM1	1.1b
	$= \frac{1}{2}\left(3\times 81-6\times 27+\frac{243}{5}\right)\left(=\frac{324}{5}\right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{64.8}{18} = 3.6$ *	A1*	2.2a
		(4)	
5a alt	$\int xy\mathrm{d}y = \int y\sqrt{9-y}\,\mathrm{d}y$	M1	2.1
	$= \left[-\frac{2}{3}y(9-y)^{\frac{3}{2}} - \frac{4}{15}(9-y)^{\frac{5}{2}}\right]_0^9$	DM1	1.1b
	$= \frac{4}{15}\times 9^{\frac{5}{2}} \quad \left(=\frac{324}{5}\right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{64.8}{18} = 3.6(\mathrm{m})$ *	A1*	2.2a
		(4)	
5b	Moments about O:	M1	3.1a
	$3.6W = 9\lambda W$	A1	1.1b
	$\lambda = 0.4$	A1	1.1b
		(3)	
			(7 marks)

Notes:	
5(a) M1	Use of $\int \frac{1}{2}y^2\mathrm{d}x$ or $\int xy\mathrm{d}y$. Ignore any limits.
DM1	Integrate and use correct limits (0 and 3 for x, 0 and 9 for y). Usual rules for integration: powers increasing by 1
A1	Correct unsimplified expression.
A1*	Obtain given answer from correct working.
5(b) M1	Complete method to obtain λ , e.g. by taking moments about O or by resolving and taking moments about a different point.

	Allow for an equation in T_A or λ
A1	Correct unsimplified equation in λ
A1	Correct only.

Question	Scheme	Marks	AOs
6			
	 Resolve vertically	M1	3.4
	$F\cos\theta + mg = R\sin\theta$ $(4F + 5mg = 3R)$	A1 A1	1.1b 1.1b
	Equation for motion towards centre	M1	3.4
	$F\sin\theta + R\cos\theta = m\times 0.5\omega^2$ $(3F + 4R = m\times 2.5\omega^2)$	A1 A1	1.1b 1.1b
	At max ω $R + 5mg = 3R \quad \left(R = \frac{5mg}{2}\right)$ $\frac{3}{4}R + 4R = m\times 2.5\omega^2 \quad (19R = 10m\omega^2)$	M1	1.2
	Solve for ω: $19\times\frac{5mg}{2} = 10m\omega^2$	DM1	3.1a
	$\omega^2 = \frac{19g}{4} \Rightarrow \max\omega = 6.8 \ (6.82\ )$	A1	2.2a
		(9)	
			(9 marks)

Notes:	
M1	Need all terms. Condone sign errors and sin/cos confusion.
A1 **A1**	Unsimplified equation with at most one error. Correct unsimplified equation.
M1	Need all terms. Condone sign errors and sin/cos confusion in *R* or their *R*.
A1 **A1**	Unsimplified equation with at most one error. Correct unsimplified equation in *R* or their *R*.
	NB: if *F* in wrong direction count this as one error (not one in each equation) and deduct one accuracy mark in the first equation affected. Either / both equation(s) could be replaced with equations for resolving parallel and perpendicular to the surface. Perpendicular: $R = mg\sin\theta + mr\omega^2\cos\theta \quad \left(R = \frac{3}{5}mg + \frac{2}{5}m\omega^2\right)$

	Parallel: $F + mg\cos\theta = mr\omega^2 \sin\theta \quad \left(F + \frac{4}{5}mg = \frac{3}{10}m\omega^2\right)$ If they have more than 2 equations, mark the correct equations. If they go on to use an incorrect equation then DM0.
M1	Use of $F = \mu R$ to eliminate F or R Condone inequality
DM1	Complete method including substitution of trig values to obtain a value for ω
A1	2 s.f or 3 s.f only $\sqrt{\frac{19g}{4}}$ is A0 Must be an equation

Question	Scheme	Marks	AOs
7(a)	Mass of dome $= \int_a^{2a} \pi y^2 \frac{\lambda}{x^2}\,\mathrm{d}x = \pi\lambda \int_a^{2a} \frac{4a^2 - x^2}{x^2}\,\mathrm{d}x = \pi\lambda \int_a^{2a} \left(\frac{4a^2}{x^2} - 1\right)\mathrm{d}x$	M1	3.4
	$= \pi\lambda \left[\frac{-4a^2}{x} - x\right]_a^{2a} \quad \left(= \pi\lambda \left[\frac{-4a^2}{2a} - 2a + \frac{4a^2}{a} + a\right] = \pi\lambda a(\text{kg})\right)$	A1	1.1b
	Moments: $\int_a^{2a} \pi y^2 \frac{\lambda}{x^2} \times x\,\mathrm{d}x = \pi\lambda \int_a^{2a} \left(\frac{4a^2}{x} - x\right)\mathrm{d}x$	M1	3.4
	$= \pi\lambda \left[4a^2 \ln x - \frac{1}{2}x^2\right]_a^{2a} \quad \left(= \pi\lambda \left(4a^2 \ln 2 - \frac{3a^2}{2}\right)\right)$	A1	1.1b
	$\Rightarrow \text{distance} = \frac{\pi\lambda \left(4a^2 \ln 2 - \frac{3a^2}{2}\right)}{\pi\lambda a}$	DM1	2.1
	$\Rightarrow \text{distance} = 4a \ln 2 - \frac{3a}{2} - a = \left(4 \ln 2 - \frac{5}{2}\right)a$ (m) *	A1*	2.2a
		(6)	
(b)	Centre of mass of cone lies a m from the plane surface	B1	1.1b
	Moments about a diameter of the plane face	M1	3.1b
	$a \times kW - a\left(4 \ln 2 - \frac{5}{2}\right) \times 2W = (2 + k)Wd$	A1	1.1b
	$d = \frac{\lvert k + 5 - 8 \ln 2 \rvert}{2 + k} a$ *	A1*	2.2a
		(4)	
(c)	Use of trigonometry	M1	3.1b
	$\tan \alpha = \frac{\left(\frac{k + 5 - 8 \ln 2}{2 + k}\right)a}{\sqrt{3}a} \left(= \frac{1}{2\sqrt{3}}\right)$	A1	1.1b
	$\Rightarrow k = 16 \ln 2 - 8$	A1	1.1b
		(3)	
			(13 marks)
Notes:			

(a)M1	Use the model to find the mass of the dome. Allow without limits.
A1	Correct integration. Correct limits seen or implied.
M1	Use the model to take moments (usual rules for integration). Allow without limits.
A1	Correct integration. Correct limits seen or implied.
DM1	Complete method to find the distance of the centre of mass from the centre of the plane face.
A1*	Obtain the given answer from full and correct working.
(b)B1	Seen or implied
M1	Moments equation. Need all terms. Dimensionally correct. Allow use of a parallel axis.
A1	Correct unsimplified equation in d
A1*	Obtain given answer from correct working. Condone if modulus signs not used
(c)M1	Complete method to find the required angle, e.g. by use of tangent. Condone if they ignore the modulus signs.
A1	Correct unsimplified equation.
A1	Any equivalent exact form.

Question	Scheme	Marks	AOs
8a			
	In equilibrium: $T_A = T_B$	M1	3.3
	$\frac{40e}{2} = \frac{20(2-e)}{2}$	A1	1.1b
	$2e = 2-e \Rightarrow e = \frac{2}{3} \Rightarrow EB = \frac{8}{3}(\text{m})$ *	A1*	2.2a
		(3)	
8b	Equation of motion for P when displaced x from equilibrium in direction of A	M1	2.1
	$\frac{40(e+x)}{2} - \frac{20(2-e-x)}{2} = -0.3\ddot{x}$	A1 A1	1.1b 1.1b
	$\ddot{x} = -100x$ Of the form $\ddot{x} = -\omega^2 x$ hence SHM*	A1*	3.2a
		(4)	
8c	Periodic time $= \frac{2\pi}{\text{their } \omega}\left(= \frac{\pi}{5}(\text{s})\right)$	M1	3.4
	$\Rightarrow S = \frac{1}{4} \times \frac{2\pi}{\text{their } \omega}$	M1	3.1a
	$S = \frac{\pi}{20}$	A1	1.1b
		(3)	
8c alt	Use of $x = a\cos(\text{their}\omega)t$ or $x = a\sin(\text{their}\omega)t$	M1	
	$0 = \frac{1}{3}\cos(\text{their}\omega)S$ or $\frac{1}{3} = \frac{1}{3}\sin(\text{their}\omega)S$	M1	
	$S = \frac{\pi}{20}$	A1	
		(3)	
8d	Use of $v = a\omega\cos\omega t$ or $v = a\omega\sin\omega t$	M1	3.4
	$2 = \frac{\omega}{3}\cos\omega t \quad (t = 0.0927...)$ or $2 = \frac{\omega}{3}\sin\omega t \quad (t = 0.06435...)$	A1ft	1.1b
	time $= 4 \times 0.0927..$ or time $= \frac{\pi}{5} - 4 \times 0.06435..$	DM1	3.1a
	time $= 0.37(\text{s}) \quad (0.371(\text{s}))$	A1	1.1b
		(4)	

	(14 marks)
Notes:	
(a) M1	Form equation for equilibrium of forces acting on *P*. Dimensionally correct.
A1	Correct unsimplified equation in one unknown.
A1*	Obtain given answer from correct working. Condone missing units.
(b) M1	Equation of motion about the equilibrium. Need all terms and dimensionally correct. Condone sign errors.
A1 A1	Unsimplified equation with *e* or given *e* with at most one error. Correct unsimplified equation with *e* or given *e*
A1*	Reach given conclusion in correct form from correct working.
(c) M1	Use the model to find the periodic time for ω obtained correctly.
M1	Correct method to find the required time for ω obtained correctly.
A1	Correct only.
(d) M1	Use a correct model for the speed or displacement of *P* for ω obtained correctly
A1ft	Follow their ω. NB $v = 2$ when $x = \frac{4}{15}$
DM1	Complete method to find the required time.
A1	0.37 or better (0.370918….)

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